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Low-Scale $SU(4)_W$ Unification

Csaba Balazs[1], **Tianjun Li**[2,3], **Fei Wang**[1], **Jin Min Yang**[2]

[1] *School of Physics, Monash University, Melbourne Victoria 3800, Australia*
[2] *Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*
[3] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*

Abstract: We embed the minimal left-right model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ into an $SU(4)_W$ gauge group, and break the unified group via five-dimensional $S^1/(Z_2 \times Z_2)$ orbifolding. Leptons are fitted into $SU(4)_W$ multiplets and located on a symmetry preserving O brane, while quarks are placed onto an O' brane where the symmetry is broken. This approach predicts $\sin^2\theta_W = 0.25$ for the weak mixing angle at tree level and leads to a rather low weakly (strongly) coupled unification scale of order 3×10^2 TeV (several TeV) with supersymmetry, or as low as several TeV in the non-supersymmetric case. Another symmetry breaking chain with the low-energy gauge group $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ can also give rise to a weak mixing angle $\sin^2\theta_W = 0.25$ at tree level after gauge symmetry breaking by orbifolding. Such theories with low-scale unification have interesting phenomenological consequences.

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1. Introduction

The standard model (SM) of electroweak interactions, based on the spontaneously broken gauge symmetry $SU(2)_L \times U(1)_Y$, has been extremely successful in describing phenomena below the weak scale. However, the SM leaves some theoretical and aesthetical questions unanswered, two of which are the origin of parity violation and the smallness of neutrino masses. Both of these questions can be addressed in the left-right model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [1]. The supersymmetric extension of this model [2] is especially intriguing since it automatically preserves R-parity. This can lead to a low energy theory without baryon number violating interactions after R-parity is spontaneously broken. However, in such left-right models parity invariance and the equality of the $SU(2)_L$ and $SU(2)_R$ gauge couplings is ad hoc and has to be put in by hand. Only in grand unified theories, based on $SO(10)$ [3], can the equality of the two $SU(2)$ gauge couplings be naturally guaranteed through gauge coupling unification. But in these grand unified theories the unification scale is usually much higher than the weak scale. For example, in the supersymmetric $SU(5)$ [4] model the weak mixing angle is predicted to be 3/8 at tree level while the measured value is 0.23 at the weak scale. The difference can only be bridged via a long renormalization evolution, which in turn requires a rather high unification scale at about 10^{16} GeV. This high-scale unification has the unsatisfactory feature that a large energy-desert lies between the weak scale and the unification scale. Therefore, it is interesting to explore the unification of the left-right symmetries at low energy scales.

Novel attempts for the unification of the left-right symmetries have been proposed in the literature, such as the $SU(4)_{PS} \times SU(4)_W$ or the $SU(4)_W \times U(1)_{B-L}$ models [5, 6, 7]. However, in these new unification models the weak mixing angle either can not be predicted (in $SU(4)_W \times U(1)_{B-L}$ the weak mixing angle is arbitrary) or predicted as 3/8 at tree level, implying a relatively high unification scale. Besides, in order to accommodate matter unification, mirror fermions are necessarily introduced in order to fill each $SU(4)_W$ multiplet. The problems of these models are similar to the difficulty in the $SU(3)_W \times U(1)$ extension [8], which uses $SU(3)_W$ to unify the SM groups $SU(2)_L \times U(1)_Y$.

With orbifold gauge symmetry breaking (OGSB) [9, 10, 11, 12, 13, 14], we can achieve gauge interaction unification while leaving matter fields partially unified or un-unified. The problem of the $SU(3)_W$ unification can be nicely tackled in this approach [15, 16, 17]. In this work, we propose the use of an $SU(4)_W$ group to unify the left-right gauge couplings on a $S^1/(Z_2 \times Z_2)$ orbifold, in which leptons are fitted into $SU(4)_W$ multiplets and located on the symmetry-preserving O brane while quarks are placed onto an O' brane with broken symmetry. This model predicts the weak mixing angle to be $\sin^2\theta_W = 0.25$ at tree level and achieves gauge coupling unification at the order of 10^2 TeV in supersymmetric cases and several TeV in non-supersymmetric cases.

The content of this work is organized as follows. In Sec. 3 we discuss $SU(4)_W$ left-right unification in the supersymmetric (SUSY) context, focusing on gauge symmetry breaking on the five-dimensional orbifold. In Sec. 4 we examine the gauge coupling running and unification, especially the compactification scale from the weak mixing angle. In Sec. 5 we discuss another $SU(4)_W$ symmetry breaking chain into $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$. Sec.

6 contains our conclusions.

2. Brief Review of Orbifold Gauge Symmetry Breaking

We consider a five-dimensional space-time $\mathcal{M}_4 \times S^1/(Z_2 \times Z_2)$ comprising of Minkowski space $\mathcal{M}_4$ with coordinates x_μ and the orbifold $S^1/(Z_2 \times Z_2)$ with the coordinate $y \equiv x_5$. The orbifold $S^1/(Z_2 \times Z_2)$ is obtained from S^1 by moduling the equivalent classes:

$$P: \; y \sim -y \; , \qquad P': \; y' \sim -y' \; , \tag{2.1}$$

with $y' \equiv y + \pi R/2$. There are two inequivalent 3-branes located at $y = 0$ and $y = \pi R/2$ which are denoted by O and O', respectively.

The five-dimensional $N = 1$ supersymmetric gauge theory has 8 real supercharges, corresponding to $N = 2$ supersymmetry in four dimensions. The vector multiplet physically contains a vector boson A_M where $M = 0, 1, 2, 3, 5$, two Weyl gauginos $\lambda_{1,2}$, and a real scalar σ. In the four-dimensional $N = 1$ language, it contains a vector multiplet $V(A_\mu, \lambda_1)$ and a chiral multiplet $\Sigma((\sigma + iA_5)/\sqrt{2}, \lambda_2)$ which transform in the adjoint representation of the gauge group. The five-dimensional hypermultiplet has two physical complex scalars ϕ and ϕ^c, a Dirac fermion Ψ, and can be decomposed into two 4-dimensional chiral multiplets $\Phi(\phi, \psi \equiv \Psi_R)$ and $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$, which transform as each others conjugates under gauge transformations.

The general action [18] for the gauge fields and their couplings to the bulk hypermultiplet Φ is

$$\begin{aligned} S = & \int d^5x \frac{1}{kg^2} \mathrm{Tr} \left[\frac{1}{4} \int d^2\theta \left(W^\alpha W_\alpha + \mathrm{h.c.} \right) \right. \\ & \left. \qquad + \int d^4\theta \left((\sqrt{2}\partial_5 + \bar{\Sigma}) e^{-V} (-\sqrt{2}\partial_5 + \Sigma) e^V + \partial_5 e^{-V} \partial_5 e^V \right) \right] \\ & + \int d^5x \left[\int d^4\theta \left(\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi \right) + \int d^2\theta \left(\Phi^c (\partial_5 - \frac{1}{\sqrt{2}} \Sigma) \Phi + \mathrm{h.c.} \right) \right] \end{aligned} \tag{2.2}$$

where $Tr(T^a T^b) = k\delta^{ab}$.

Because the action is invariant under the parity operation P, under this operation, the vector multiplet transforms as

$$V(x^\mu, y) \to V(x^\mu, -y) = PV(x^\mu, y)P^{-1} \; , \tag{2.3}$$

$$\Sigma(x^\mu, y) \to \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1} \; . \tag{2.4}$$

If the hypermultiplet belongs to the fundamental or anti-fundamental representations, since $P = P^{-1}$, we have

$$\Phi(x^\mu, y) \to \Phi(x^\mu, -y) = \eta_\Phi P\Phi(x^\mu, y) \; , \tag{2.5}$$

$$\Phi^c(x^\mu, y) \to \Phi^c(x^\mu, -y) = -\eta_\Phi P\Phi^c(x^\mu, y) \; . \tag{2.6}$$

Alternatively, if the hypermultiplet belongs to the symmetric, anti-symmetric or adjoint representations, we have

$$\Phi(x^\mu, y) \to \Phi(x^\mu, -y) = \eta_\Phi P \Phi(x^\mu, y) P \ , \tag{2.7}$$

$$\Phi^c(x^\mu, y) \to \Phi^c(x^\mu, -y) = -\eta_\Phi P \Phi^c(x^\mu, y) P \ , \tag{2.8}$$

where $\eta_\Phi = \pm 1$.

Similar results hold for the parity operation P', we just need to make the following replacements in the above equations:

$$P \longrightarrow P' \ , \qquad \eta_\Phi \longrightarrow \eta'_\Phi \ . \tag{2.9}$$

The gauge symmetry and supersymmetry can be broken by choosing suitable representations for P and P'. For a field ϕ, in the representation of unbroken gauge symmetry, we obtain the following transformation

$$\phi(x_\mu, y) \to \phi(x_\mu, -y) = p_\phi \phi(x_\mu, y) \ , \tag{2.10}$$

$$\phi(x_\mu, y') \to \phi(x_\mu, -y') = p'_\phi \phi(x_\mu, y') \ , \tag{2.11}$$

where $p_\phi = \pm 1$ and $p'_\phi = \pm 1$. Introducing the notation $\phi_{p_\phi p'_\phi}$, we obtain the Kaluza-Klein (KK) mode expansions as of such ϕ fields as follows

$$\phi_{++}(x_\mu, y) = \sum_{n=0}^{+\infty} \sqrt{\frac{1}{2^{\delta_{n,0}} \pi R}} \phi_{++}^{(2n)}(x_\mu) \cos \frac{2ny}{R} \ , \tag{2.12}$$

$$\phi_{+-}(x_\mu, y) = \sum_{n=0}^{+\infty} \sqrt{\frac{1}{\pi R}} \phi_{+-}^{(2n+1)}(x_\mu) \cos \frac{(2n+1)y}{R} \ , \tag{2.13}$$

$$\phi_{-+}(x_\mu, y) = \sum_{n=0}^{+\infty} \sqrt{\frac{1}{\pi R}} \phi_{-+}^{(2n+1)}(x_\mu) \sin \frac{(2n+1)y}{R} \ , \tag{2.14}$$

$$\phi_{--}(x_\mu, y) = \sum_{n=0}^{+\infty} \sqrt{\frac{1}{\pi R}} \phi_{--}^{(2n+2)}(x_\mu) \sin \frac{(2n+2)y}{R} \ . \tag{2.15}$$

Here n is an integer and the fields $\phi_{++}^{(2n)}(x_\mu)$, $\phi_{+-}^{(2n+1)}(x_\mu)$, $\phi_{-+}^{(2n+1)}(x_\mu)$ and $\phi_{--}^{(2n+2)}(x_\mu)$ respectively acquire a mass of $2n/R$, $(2n+1)/R$, $(2n+1)/R$ and $(2n+2)/R$ upon compactification. Only $\phi_{++}(x_\mu, y)$ possesses a four-dimensional massless zero mode. It is easy to see that ϕ_{++} and ϕ_{+-} are non-vanishing at $y = 0$, while ϕ_{++} and ϕ_{-+} are non-vanishing at $y = \pi R/2$.

3. $SU(4)_W$ Unification of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

In the minimal left-right model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the discrete symmetry ensuring identical $SU(2)_L$ and $SU(2)_R$ gauge couplings is put in by hand. In this work we use $SU(4)_W$ to unify the left-right symmetries and interpret the origin of the parity

invariance as the residual discrete symmetry from the symmetry breaking of the unification group. Since we intend to truly unify the left-right gauge groups, it is not possible to fill all the matter fields into $SU(4)_W$ multiplets without introducing mirror fermions because of the different $U(1)_{B-L}$ charge assignments for quarks and leptons. So we opt for the OGSB mechanism with partial unification for matter fields.

Starting from the five-dimensional $SU(3)_C \times SU(4)_W$ gauge theory, we can choose the following Z_2 matrix representations for P and P' in the adjoint representation of $SU(3)_C \times SU(4)_W$:

$$P = \mathrm{diag}(+1,+1,+1) \otimes \mathrm{diag}(+1,+1,+1,+1) \ , \tag{3.1}$$

$$P' = \mathrm{diag}(+1,+1,+1) \otimes \mathrm{diag}(+1,+1,-1,-1) \ . \tag{3.2}$$

The gauge symmetry $SU(4)_W$ is broken by boundary conditions to $SU(2)_L \times SU(2)_R \times U(1)_X$ on the boundary O' brane while is preserved in the bulk and on the O brane. Consequently, the parity assignments for V and Σ are

$$V(\mathbf{15}) = (\mathbf{3},\mathbf{1})_{\mathbf{0}}^{(+,+)} \oplus (\mathbf{1},\mathbf{3})_{\mathbf{0}}^{(+,+)} \oplus (\mathbf{2},\bar{\mathbf{2}})_{\mathbf{2}}^{(+,-)} \oplus (\bar{\mathbf{2}},\mathbf{2})_{-\mathbf{2}}^{(+,-)} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{0}}^{(+,+)} \ , \tag{3.3}$$

$$\Sigma(\mathbf{15}) = (\mathbf{3},\mathbf{1})_{\mathbf{0}}^{(-,-)} \oplus (\mathbf{1},\mathbf{3})_{\mathbf{0}}^{(-,-)} \oplus (\mathbf{2},\bar{\mathbf{2}})_{\mathbf{2}}^{(-,+)} \oplus (\bar{\mathbf{2}},\mathbf{2})_{-\mathbf{2}}^{(-,+)} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{0}}^{(-,-)} \ . \tag{3.4}$$

We place the lepton sector on the O brane while keep the quark sector on the O' brane. This means that only the leptons are filled into $SU(4)_W$ multiplets

$$(\mathbf{4}): \quad L = \mathrm{diag}\left(\nu_L \ , \ e_L \ , \ e_L^c \ , \ -\nu_L^c\right) \ . \tag{3.5}$$

Here $\phi_L^c \equiv (\phi^c)_L$ and the minus sign conforms to our choice of $Q^a = (\nu_L^c, e_L^c)$ in $SU(2)_R$ representations $\bar{\mathbf{2}}$ and being related to its conjugate by $Q_a = (e_L^c, -\nu_L^c)$ through the fully antisymmetric tensor $Q^a = \epsilon^{ab} Q_b$. From the $SU(4)_W$ fundamental representation and its proper normalization follows that the $U(1)_X$ charge assignment of the fundamental representation can be written as

$$Y_X = \mathrm{diag}\left(-1,-1,\ 1,\ 1\right) \ . \tag{3.6}$$

From this we can identify $U(1)_X$ as $U(1)_{B-L}$. The normalization of the gauge group $U(1)_{B-L}$ reads

$$T_{B-L} = \frac{\sqrt{2}}{2}\frac{Y_{B-L}}{2} \ . \tag{3.7}$$

From the normalization condition follows the relation between the gauge couplings of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(4)_W$

$$g_{B-L}^2 = \frac{1}{2} g_4^2 \ , \qquad g_L^2 = g_R^2 = g_4^2 \ , \tag{3.8}$$

which holds at the $SU(4)_w$ unification scale. Hence we can predict the tree-level weak mixing angle as

$$\sin^2\theta_W = \frac{g_{B-L}^2}{g_L^2 + 2g_{B-L}^2} = \frac{1}{4} \ . \tag{3.9}$$

To induce Yukawa couplings for the $SU(4)_W$ multiplet leptons, we can introduce bulk Higgs fields in the $SU(4)_W$ antisymmetric representation $\Phi_{ab}(\bar{\mathbf{6}})$ and symmetric representations $\Delta^i_{ab}(\mathbf{10})$ and $\Delta^i_{ab}(\overline{\mathbf{10}})$ [1]. In four dimensions there are eight $N=1$ chiral multiplets $\Phi, (\Phi^c)$, Δ^i, $(\Delta^c)^i$ $(i=1,\ 2,\ 3)$. We assign the boundary conditions for the Higgs fields as

$$\eta_\Phi = 1,\quad \eta'_\Phi = -1,\quad \eta_{\Delta^1} = 1,\quad \eta'_{\Delta^1} = -1,\quad \eta_{\Delta^i} = 1,\quad \eta'_{\Delta^i} = 1\ (i=2,3) \tag{3.10}$$

The parities of the Higgs fields in terms of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ representation are given by

$$\begin{aligned}
\Phi(\bar{\mathbf{6}}) &= (\mathbf{1},\mathbf{1})^{(+-)}_{\mathbf{2}} \oplus (\mathbf{1},\mathbf{1})^{(+-)}_{-\mathbf{2}} \oplus (\bar{\mathbf{2}},\bar{\mathbf{2}})^{(++)}_{\mathbf{0}}\ ,\\
\Delta^1(\overline{\mathbf{10}}) &= (\bar{\mathbf{3}},\mathbf{1})^{(+-)}_{-\mathbf{2}} \oplus (\mathbf{1},\bar{\mathbf{3}})^{(+-)}_{\mathbf{2}} \oplus (\bar{\mathbf{2}},\bar{\mathbf{2}})^{(++)}_{\mathbf{0}}\ ,\\
\Delta^2(\overline{\mathbf{10}}) &= (\bar{\mathbf{3}},\mathbf{1})^{(++)}_{-\mathbf{2}} \oplus (\mathbf{1},\bar{\mathbf{3}})^{(++)}_{\mathbf{2}} \oplus (\bar{\mathbf{2}},\bar{\mathbf{2}})^{(+-)}_{\mathbf{0}}\ ,\\
\Delta^3(\mathbf{10}) &= (\mathbf{3},\mathbf{1})^{(++)}_{\mathbf{2}} \oplus (\mathbf{1},\mathbf{3})^{(++)}_{-\mathbf{2}} \oplus (\mathbf{2},\mathbf{2})^{(+-)}_{\mathbf{0}}\ .
\end{aligned} \tag{3.11}$$

Under these parity assignments the conjugate chiral fields (Φ^c), $(\Delta^c)^i$ $(i=1,\ 2,\ 3)$ have no zero modes and irrelevant to the low energy phenomenology. The zero modes form two $SU(2)_L$ and $SU(2)_R$ triplets with opposite $U(1)_{B-L}$ quantum numbers and two bi-doublets $(2,2)$ with vanishing $U(1)_{B-L}$ quantum numbers which give exactly the Higgs field contents of the supersymmetric left-right model.

As the leptons are fitted into the $SU(4)_W$ multiplets, we can write down their Yukawa interactions with the bulk Higgs fields. Since the leptons are placed on the O brane, it is obvious that they are invariant under P transformation. The transformation property for the leptons under P' is determined by the requirement that the operators on the O brane must transform covariantly under P', otherwise the gauge symmetry preserved at $y=0$ will not be preserved at the $y=\pi R$ brane. From the kinetic terms of the leptons we can get the transformation property of the leptons under P' as

$$P' : (\nu_L\ e_L\ e^c_L\ -\nu^c_L) \to \pm(+,+,-,-)\ . \tag{3.12}$$

The transformation of the Yukawa interactions under P' is

$$P' :\ \sum_{ij} Y_{ij} L^a_{[i} L^b_{j]} \Phi_{[ab]}(\bar{\mathbf{6}}) = -\ , \tag{3.13}$$

where (i,j) is antisymmetric, and

$$P' :\ \sum_{ij} Y_{ij} L^a_i L^b_j \Delta^1_{ab}(\overline{\mathbf{10}}) = -\ , \tag{3.14}$$

$$P' :\ \sum_{ij} Y_{ij} L^a_i L^b_j \Delta^2_{ab}(\overline{\mathbf{10}}) = +\ , \tag{3.15}$$

[1] If we introduce only $\Phi_{ab}(\bar{\mathbf{6}})$ to give charged lepton masses, the allowed Yukawa couplings have the form $y_{[ij]} L^i_{[a} L^j_{b]} \Phi^{[ab]}$ with index ab(and ij) being antisymmetric. Such Yukawa couplings lead to $m_e = 0$ and $m_\mu = m_\tau$, which is unrealistic. Note that it is also possible to introduce four Higgs hypermultiplets in the symmetric representation $\Delta^i_{ab}(\mathbf{10})$ and $\Delta^i_{ab}(\overline{\mathbf{10}})$ in this scenario.

where i, j are the family indices. So we can write the Yukawa interactions as

$$\mathcal{L}_5 = \int d^2\theta\sqrt{\pi R}\left[\frac{1}{2}\left\{\delta(y)-\delta(y-\pi R)\right\}\sum_{ij}\left(Y^1_{[ij]}L^a_i L^b_j \Phi^1_{[ab]} + Y^2_{ij}L^a_i L^b_j \Delta^1_{ab}\right)\right.$$
$$\left. +\frac{1}{2}\left\{\delta(y)+\delta(y-\pi R)\right\}\sum_{ij} Y^3_{ij} L^a_i L^b_j \Delta^2_{ab}\right] \ . \tag{3.16}$$

After integrating out the fifth dimensional coordinate, we get the Yukawa couplings in four dimensions

$$\mathcal{L}_4 = \sum_{n=0}^{\infty}\int d^2\theta \sum_{ij}\Big[\ \frac{1}{\sqrt{2^{n,0}}}\big(\ y^{1[ij]}L_{[i}L^c_{j]}\phi_1^{(2n)} + y^{2ij}L_i L^c_j \phi_2^{(2n)}$$
$$+\, y^{3ij}L_i L_j \Delta_1^{(2n)} + y^{4ij}(L^c_i)L^c_j\Delta_2^{(2n)}\ \big)\ \Big] + h.c.\ , \tag{3.17}$$

where the lepton $SU(4)_W$ multiplets are decomposed as $\mathbf{4} = (L\ L^c)$, the bi-doublet Higgs fields ϕ_1 and ϕ_2 belong to the $(2,2)_0$ representations of $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, and the triplets Δ_1 and Δ_2 belong to $(3,1)_{-2}$ and $(1,3)_2$ representations. The interactions for the zero modes of the Higgs fields are the Yukawa couplings in the supersymmetric left-right model. Similarly, we can write the couplings of the lepton multiplets with the vector multiplet V^a and the chiral multiplet Σ^a.

Supersymmetry breaking can be realized via the Scherk-Schwarz mechanism through the boundary conditions [21, 22, 23]. It is well known that $N=1$ supersymmetry in five dimensions possesses an $SU(2)_R$ global R-symmetry under which the gauginos from the vector multiplets (λ_1, λ_2) and complex scalars $(\phi, \phi^{c\dagger})$ from hypermultiplets form $SU(2)_R$ doublets. The non-trivial twist T for translation with respect to $SU(2)_R$ R-symmetry can be written as [22, 23]

$$T = \exp\left(-2\pi i\sigma_2\alpha\right)\ , \tag{3.18}$$

with orbifolding projection

$$P' = \sigma_3\ . \tag{3.19}$$

Besides, the symmetric Higgs bosons $\Delta^i(\mathbf{10})$(i=1,2,3) have a $SU(3)$ flavor symmetry. Also, the consistent relation between the translation and the orbifolding is

$$TP'T = P'\ , \tag{3.20}$$

where P' is the reflection according to Z_2(or Z'_2), and T is the translation

$$T\phi(x_\mu, y) = \phi(x_\mu, y+2\pi R)\ . \tag{3.21}$$

We denote the translation operator T corresponding to the global $SU(3)$ flavor symmetry as follows

$$T = \exp\left(2\pi i\sum_a T^a\theta^a\right)\ , \tag{3.22}$$

where T^a are $SU(3)$ generators. From formula (3.20), we obtain

$$\left\{T^a\theta^a, P'\right\} = 0 \ . \tag{3.23}$$

For the following non-trivial 3×3 matrix

$$P' = \begin{pmatrix} \pm 1 & 0 \\ 0 & \sigma_3 \end{pmatrix} \ , \tag{3.24}$$

the most general form of T can be described by

$$T = \exp\left[2\pi i(\gamma_0 T^0 + \gamma_1 T^1 + \gamma_2 T^2)\right] \ , \tag{3.25}$$

with

$$T^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \ , \qquad T^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \ , \tag{3.26}$$

$$T^2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} (P'_{11} = 1) \ \text{ or } \ T^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} (P'_{11} = -1) \ . \tag{3.27}$$

The parameter γ_0 can be rotated away by the residue global symmetry, so the twist for flavor $SU(3)$ compatible with non-trivial P' can be written as

$$T = \exp\left[2\pi i(\gamma_1 T^1 + \gamma_2 T^2)\right] \ . \tag{3.28}$$

In case of the $SU(3)$ flavor symmetry for $\Delta^1(\mathbf{\bar{10}}), \Delta^2(\mathbf{\bar{10}}), \Delta^3(\mathbf{10})$, the relative parity assignments under P and P' are nontrivial with

$$P = \begin{pmatrix} 1 & 0 \\ 0 & \sigma_3 \end{pmatrix} \ , \ P' = \begin{pmatrix} -1 & 0 \\ 0 & \sigma_3 \end{pmatrix} \ . \tag{3.29}$$

So the twist boundary condition T compatible with both are

$$T = \exp\left(2\pi i\gamma_1 T^1\right) \ . \tag{3.30}$$

The most general boundary conditions for the fields are

$$A^M(x^\mu, y + 2\pi R) = A^M(x^\mu, y) \ , \tag{3.31}$$

$$\sigma(x^\mu, y + 2\pi R) = \sigma(x^\mu, y) \ , \tag{3.32}$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}(x^\mu, y + 2\pi R) = e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}(x^\mu, y) \ , \tag{3.33}$$

$$\begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix}(x^\mu, y + 2\pi R) = e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix}(x^\mu, y) \ , \tag{3.34}$$

$$\begin{pmatrix} \tilde{\delta}_1 & \tilde{\delta}_2 & \tilde{\delta}_3 \\ \tilde{\delta}_1^{c\dagger} & \tilde{\delta}_2^{c\dagger} & \tilde{\delta}_3^{c\dagger} \end{pmatrix}(x^\mu, y + 2\pi R) = \begin{pmatrix} \tilde{\delta}_1 & \tilde{\delta}_2 & \tilde{\delta}_3 \\ \tilde{\delta}_1^{c\dagger} & \tilde{\delta}_2^{c\dagger} & \tilde{\delta}_3^{c\dagger} \end{pmatrix} e^{2\pi i\gamma_1 T^1}(x^\mu, y) \ , \tag{3.35}$$

$$\begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_1^{c\dagger} & \delta_2^{c\dagger} & \delta_3^{c\dagger} \end{pmatrix}(x^\mu, y + 2\pi R) = e^{-2\pi i\alpha\sigma_2}\begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_1^{c\dagger} & \delta_2^{c\dagger} & \delta_3^{c\dagger} \end{pmatrix} e^{2\pi i\gamma_1 T^1}(x^\mu, y) \ . \tag{3.36}$$

Here we denote the components of chiral supermultiplets $\Delta^i(\mathbf{10})$ as $(\delta^i(\mathbf{10}), \tilde{\delta}^i(\mathbf{10}))$ with their conjugate chiral supermultiplets $\Delta^{ic}(\overline{\mathbf{10}})$ as $(\delta^{ic}(\mathbf{10}), \tilde{\delta}^{ic}(\mathbf{10}))$. The complex scalar components for hypermultiplets $(\Phi(\mathbf{6}), \Phi^c(\overline{\mathbf{6}}))$ are denoted as $(\phi, \phi^{c\dagger})$.

We now consider the modes expansion of the fields with respect to the previous Scherk-Schwarz type boundary conditions. For simplicity, we write explicitly only the relative P and P' parity assignments under orbifolding projections

$$\begin{pmatrix} \lambda_1^{(++)} \\ \lambda_2^{(--)} \end{pmatrix}(x^\mu, y) = \sum_{n=0}^{\infty} e^{-i\alpha\sigma_2 y/R} \begin{pmatrix} \sqrt{\frac{1}{2^{\delta_{n,0}}\pi R}}(\lambda_1^{(++)})^{(2n)}(x_\mu)\cos\frac{2ny}{R} \\ \sqrt{\frac{1}{\pi R}}(\lambda_2^{(--)})^{(2n+2)}(x_\mu)\sin\frac{(2n+2)y}{R} \end{pmatrix}, \tag{3.37}$$

$$\begin{pmatrix} \phi^{(++)} \\ (\phi^{c\dagger})^{(--)} \end{pmatrix}(x^\mu, y) = \sum_{n=0}^{\infty} e^{-i\alpha\sigma_2 y/R} \begin{pmatrix} \sqrt{\frac{1}{2^{\delta_{n,0}}\pi R}}\phi_{1++}^{(2n)}(x_\mu)\cos\frac{2ny}{R} \\ \sqrt{\frac{1}{\pi R}}\phi_{2--}^{(2n+2)}(x_\mu)\sin\frac{(2n+2)y}{R} \end{pmatrix}, \tag{3.38}$$

$$\begin{pmatrix} \tilde{\delta}_1^{+-} & \tilde{\delta}_2^{++} & \tilde{\delta}_3^{--} \\ \tilde{\delta}_1^{c\dagger-+} & \tilde{\delta}_2^{c\dagger--} & \tilde{\delta}_3^{c\dagger++} \end{pmatrix}(x^\mu, y) = \sum_{n=0}^{\infty} \begin{pmatrix} (\tilde{\delta}_1)^{+-}_{(2n)} & (\tilde{\delta}_2)^{++}_{(2n)} & (\tilde{\delta}_3)^{--}_{(2n+2)} \\ (\tilde{\delta}_1^{c\dagger})^{-+}_{(2n+2)} & (\tilde{\delta}_2^{c\dagger})^{--}_{(2n+2)} & (\tilde{\delta}_3^{c\dagger})^{++}_{(2n)} \end{pmatrix} e^{i\gamma_1 T^1 y/R}(x^\mu, y)\ , \tag{3.39}$$

$$\begin{pmatrix} \delta_1^{+-} & \delta_2^{++} & \delta_3^{--} \\ \delta_1^{c\dagger-+} & \delta_2^{c\dagger--} & \delta_3^{c\dagger++} \end{pmatrix}(x^\mu, y) = \sum_{n=0}^{\infty} e^{-i\alpha\sigma_2 y/R} \begin{pmatrix} (\delta_1)^{+-}_{(2n)} & (\delta_2)^{++}_{(2n)} & (\delta_3)^{--}_{(2n+2)} \\ (\delta_1^{c\dagger})^{-+}_{(2n+2)} & (\delta_2^{c\dagger})^{--}_{(2n+2)} & (\delta_3^{c\dagger})^{++}_{(2n)} \end{pmatrix} e^{i\gamma_1 T^1 y/R}(x^\mu, y)\ , \tag{3.40}$$

in which we represent the symmetric $\Delta^i(\overline{\mathbf{10}})$(i=1,2,3) by its $(\overline{\mathbf{3}}, \mathbf{1})_{-2}$ modes. The zero modes from the orbifold projection can get mass terms from the previous Scherk-Schwarz boundary conditions

$$\begin{aligned}\mathcal{L} = &-\frac{\alpha}{2R}\sum_a(\lambda_0^a\lambda_0^a + h.c.) - \frac{\alpha}{R}\left(\tilde{\Delta}_L^1\tilde{\Delta}_L^2 + \tilde{\Delta}_R^1\tilde{\Delta}_R^2 + h.c.\right) - \frac{\alpha^2}{R}\left(Tr\left(\Phi_1^\dagger\Phi_1\right) + Tr\left(\Phi_2^\dagger\Phi_2\right)\right) \\ &-\left(\frac{\alpha^2}{R^2} + \frac{\gamma^2}{R^2}\right)\left(Tr\left(\Delta_L^{1\dagger}\Delta_L^1\right) + Tr\left(\Delta_L^{2\dagger}\Delta_L^2\right) + Tr\left(\Delta_R^{1\dagger}\Delta_R^1\right) + Tr\left(\Delta_R^{2\dagger}\Delta_R^2\right)\right) \\ &+\frac{2\alpha\gamma}{R^2}\left(\Delta_L^1\Delta_L^2 + \Delta_R^1\Delta_R^2 + h.c.\right)\ .\end{aligned} \tag{3.41}$$

Here triplets $\Delta_L^1\left[(\overline{\mathbf{3}}, \mathbf{1})_{(-2)}\right], \Delta_R^1\left[(\mathbf{1}, \overline{\mathbf{3}})_{(2)}\right]$ are zero modes from $\Delta^2(\overline{\mathbf{10}})$ while triplets $\Delta_L^2\left[(\mathbf{3}, \mathbf{1})_{(2)}\right], \Delta_R^2\left[(\mathbf{1}, \mathbf{3})_{(-2)}\right]$ from $\Delta^3(\mathbf{10})$. The bi-doublets $\Phi_1(\overline{\mathbf{2}}, \overline{\mathbf{2}})_0$ are zero modes from $\Phi(\overline{\mathbf{6}})$ while $\Phi_2(\overline{\mathbf{2}}, \overline{\mathbf{2}})_0$ from $\Delta^1(\overline{\mathbf{10}})$. The gauge index a runs over the left-right gauge group $SU(3)_c$, $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$. The continuous parameters α and γ can be chosen to be $\alpha, \gamma \ll 1$ [23] or $\alpha, \gamma \sim \mathcal{O}(1)$ [22]. We chose the former case with $\alpha, \gamma \ll 1$. If

the scale of the supersymmetry breaking soft mass terms α/R and γ/R is chosen to be at the order of the electroweak scale, we can get the relation $M_S < M_R$. Otherwise if the scale for the supersymmetry breaking soft mass terms is higher than M_R which will not explain the gauge hierarchy problem, $M_R < M_S$ is also possible. In our case the matter contents are placed at the orbifold fix point so that no tree-level mass terms are generated through orbifolding. However, the sfermions masses can be radiatively generated through renormalization group equations below the compactification scale. Since such interactions are almost flavor universal, the supersymmetric flavor problems can be solved.There is a lot of freedom to tune the complicate Higgs potential to break the left-right symmetry down to $U(1)_Q$ directly. In supersymmetric left-right models, sneutrinos can couple to the Higgs sector which leads to spontaneously broken R-parity if such sneutrino doublets acquire vacuum expectation values. The couplings between the triplets and sneutrino which arise from the Yukawa superpotential are rather arbitrary. Detailed discussion on Higgs potential coupled to sneutrino doublets can be found in Ref. [2, 20] which will not be discussed here. In SUSY left-right cases, R-parity is automatically conserved.

The chiral anomaly cancellation in OGSB cases has been studied in [25, 26, 27, 28]. In our case with $S^1/(Z_2 \times Z_2)$ OGSB, if the gauge anomaly in four dimensions is cancelled, the five-dimensional fix-point gauge anomaly can be cancelled by introducing appropriate bulk Chern-Simons terms with jumping coefficients. At the fix point O, the gauge anomaly from the lepton $\mathbf{4}$ representation and the bulk Higgsinos in representation $\bar{\mathbf{6}}, \overline{\mathbf{10}}, \mathbf{10}$ are cancelled by the five-dimensional Chern-Simons terms. Such Chern-Simons terms also cancel the quark contribution on the O' brane. At the fix point O' we can see that the four-dimensional anomaly associated with the bulk Higgsinos is cancelled automatically although the bulk fermion contributions to the anomaly associated with the unbroken gauge group add up.

Alternative Models: It is also possible to put the leptons into the bulk by introducing mirror leptons and placing quarks on the broken symmetry O' brane. We can introduce bulk hypermultiplets (F_L, F_R) in the $(\mathbf{1}, \mathbf{4})$ representation and (F_L^c, F_R^c) in the $(\mathbf{1}, \bar{\mathbf{4}})$ representation. These multiplets are filled as:

$$F_L = (L_L, X_L) \ , \qquad F_R = (X_L^c, L_L^c) \ , \tag{3.42}$$

$$F_L^c = (\bar{L}_L, \bar{X}_L) \ , \qquad F_R^c = (\bar{X}_L^c, \bar{L}_L^c) \ , \tag{3.43}$$

where $X_L^c, \bar{L}_L, \bar{X}_L^c (X_L, \bar{L}_L^c, \bar{X}_L)$are left (right) handed mirror leptons. L_L and L_L^c are left and right handed leptons in minimal left right model, respectively. Lepton doublets in the minimal left-right model can be obtained by introducing the following parity assignments:

$$\eta_{F_L} = 1 \ , \quad \eta'_{F_L} = 1 \ , \quad \eta_{F_R} = +1 \ , \quad \eta'_{F_R} = -1 \ . \tag{3.44}$$

Lepton $SU(2)_L$ doublets survive projections from F_L while lepton $SU(2)_R$ doublets from F_R. We can see from the charge assignments of the bulk hypermultiplets that the tree level weak mixing angle $\sin^2\theta_W = 0.25$ still holds in this scenario.

The Higgs sector can be placed in the bulk or localized on the broken symmetry O' brane. In the latter case, we need two bi-doublets $(\mathbf{2}, \mathbf{2}, \mathbf{0})$, two $SU(2)_L$ triplets $(\mathbf{3}, \mathbf{1}, \pm\mathbf{2})$

and two $SU(2)_R$ triplets $(\mathbf{1}, \mathbf{3}, \pm\mathbf{2})$ in left-right gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ representations. The case of the bulk Higgs is almost identical to that of the previous case, so we do not discuss it in detail.

It is also possible to put quarks in the bulk while locate leptons on the O' brane. Then we introduce mirror quarks $\bar{Q}, \bar{Q}^c$ to fill $SU(3)_c \times SU(4)_W$ representations as:

$$(\mathbf{3}, \mathbf{4}) = \mathrm{diag}(U_L \ , \ \bar{Q}_L^c) \ , \qquad (\bar{\mathbf{3}}, \mathbf{4}) = \mathrm{diag}(\bar{Q} \ , \ U_L^c) \ , \tag{3.45}$$

where $U_L^c = (d_L^c, -u_L^c)$ denote the $\mathbf{2}$ representations in $SU(2)_R$. In this case the $U(1)_{B-L}$ charge for $SU(4)_W$ fundamental representation reads

$$Y_{B-L} = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}) \ , \tag{3.46}$$

which is normalized with respect to the $SU(4)_W$ generator T_{B-L} as:

$$T_{B-L} = \frac{3\sqrt{2}}{2} \frac{Y_{B-L}}{2} \ . \tag{3.47}$$

From the gauge coupling relations

$$g_{B-L} = \frac{3\sqrt{2}}{2} g_4 \ , \qquad g_L = g_R = g_4 \ , \tag{3.48}$$

we can get the tree level weak mixing angle

$$\sin^2\theta_W = \frac{g_{B-L}^2}{g_L^2 + 2g_{B-L}^2} = 0.45 \ , \tag{3.49}$$

which is not acceptable as a low-energy unification model.

4. Gauge Coupling Running and Unification Scale

In this section we discuss the renormalization group equation (RGE) running of the gauge couplings in the orbifold breaking case. We consider only the simplest scenario without mirror fermions. At the weak scale our inputs are [19]

$$M_Z = 91.1876 \pm 0.0021 \ , \tag{4.1}$$
$$\sin^2\theta_W(M_Z) = 0.2312 \pm 0.0002 \ , \tag{4.2}$$
$$\alpha_{em}^{-1}(M_Z) = 127.906 \pm 0.019 \ , \tag{4.3}$$
$$\alpha_3(M_z) = 0.1187 \pm 0.0020 , \tag{4.4}$$

which fix the numerical values of the standard $U(1)_Y$ and $SU(2)_L$ couplings at the weak scale

$$\alpha_1(M_Z) = \frac{\alpha_{em}(M_Z)}{\cos^2\theta_W} = (98.3341)^{-1} \ , \tag{4.5}$$
$$\alpha_2(M_Z) = \frac{\alpha_{em}(M_Z)}{\sin^2\theta_W} = (29.5718)^{-1} \ . \tag{4.6}$$

The RGE running of the gauge couplings reads

$$\frac{d\ \alpha_i}{d\ln E} = \frac{b_i}{2\pi}\alpha_i^2 \ , \tag{4.7}$$

where E is the energy scale and b_i are the beta functions. At the scale of the $SU(2)_R$ gauge boson mass M_R, the left-right $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry breaks to the SM gauge group. From the symmetry breaking chain and the normalization of the gauge field $(g_{B-L}Y_{B-L}/2)A_\mu^{B-L}$ in the kinetic term, we obtain the relation

$$\frac{1}{e^2} = \frac{1}{g_{2L}^2} + \frac{1}{g_{2R}^2} + \frac{1}{g_{B-L}^2} \, , \tag{4.8}$$

from which we can calculate the coupling g_{B-L} at the scale M_R.

Note that in non-supersymmetric left-right models neutrino masses arise by a Type I or Type II see-saw mechanism. In this case an $\mathcal{O}(\text{TeV})$ mass is unnatural for the W_R gauge boson due to the mixing term $\text{Tr}(\Phi\Delta_L\Phi^\dagger\Delta_R^\dagger)$. In the supersymmetric left-right model such a mixing term is not allowed by supersymmetry and thus a TeV-scale W_R mass is realistic [20]. We know that we need two bi-doublets to give tree-level Cabibbo-Kobayashi-Maskawa (CKM) mixings in supersymmetric left-right models. Thus, in the low energy limit, the electroweak symmetry breaking Higgs sector is non-minimal, containing two bi-doublets [2]. The corresponding supersymmetric extension (below M_R) also contains two bi-doublets.

Assuming that the $SU(2)_R$ gauge boson mass M_R is in the range 1 TeV $< M_R < M_C$ (where M_C is the compactification scale) and the mass of its superpartner falls in the range 200 GeV $< M_S < M_C$, we have two possibilities:

(i) One possibility is that $M_S < M_R$. In this case $\alpha, \gamma \ll 1$ with $\alpha/R, \gamma/R$ at the order of the electro-weak scale. Then the beta functions for the gauge couplings of $U(1)_Y, SU(2)_L, SU(3)_c$ are given by

$$(b_1, b_2, b_3) = \left(\frac{22}{3}, -\frac{8}{3}, -7\right) \quad \text{for } M_Z < E < M_S \ , \tag{4.9}$$

$$(b_1, b_2, b_3) = (\ 12,\ 2, -3) \qquad \text{for } M_S < E < M_R \ , \tag{4.10}$$

while for $\sqrt{2}U(1)_{B-L}, SU(2)_L, SU(2)_R$, and $SU(3)_c$ they are given by

$$(b_1, b_2^L, b_2^R, b_3) = (\ 8,\ 6,\ 6, -3) \qquad \text{for } M_R < E < M_C \ . \tag{4.11}$$

(ii) The other possibility is $M_R < M_S$. In this case $\alpha/R, \gamma/R$ are of order $\mathcal{O}(10)$ TeV and the gauge hierarchy problem is not solved by the high energy supersymmetry. Then we have

$$(b_1, b_2, b_3) = \left(\frac{22}{3}, -\frac{8}{3}, -7\right) \qquad \text{for } M_Z < E < M_R \ , \tag{4.12}$$

$$(b_1, b_2^L, b_2^R, b_3) = \left(\frac{10}{3}, -\frac{4}{3}, -\frac{4}{3}, -7\right) \quad \text{for } M_R < E < M_S \ , \tag{4.13}$$

$$(b_1, b_2^L, b_2^R, b_3) = (8, 6, 6, -3) \qquad \text{for } M_S < E < M_C \ . \tag{4.14}$$

[2]Due to the left-right symmetry, the left-handed triplets have the same masses as the right-handed triplets which are at the order of the M_R scale. On the other hand, the vacuum expectation values (VEVs) for the left-handed triplets are small because such VEVs will break $SU(2)_L$.

Above the SUSY left-right scale the RGE running of the gauge couplings receives contributions from KK modes

$$\alpha_i^{-1}(E) = \alpha_i^{-1}(M_R) + \frac{b_i}{2\pi}\ln\left(\frac{M_R}{E}\right) + \frac{b_{i,e}}{2\pi}\sum_{n=1}^{k}\ln\left(\frac{2n}{ER}\right)\Theta(E-\frac{2n}{R}) + \frac{b_{i,o}}{2\pi}\sum_{n=0}^{k}\ln\left(\frac{2n+1}{ER}\right)\Theta(E-\frac{2n+1}{R}). \tag{4.15}$$

Here $\Theta(x)$ is the step function defined as $\Theta(x)=1$ for $x\geq 0$ and $\Theta(x)=0$ for $x<0$. The beta functions corresponding to the even and odd KK modes at 1-loop are

$$(b_{B-L,e}, b_{B-L,o}) = (12,\ 0)\ , \tag{4.16}$$
$$(b_{2,e}^L, b_{2,o}^L) = (\ 8,\ 4)\ , \tag{4.17}$$
$$(b_{2,e}^R, b_{2,o}^R) = (\ 8,\ 4)\ , \tag{4.18}$$
$$(b_{3,e}^R, b_{3,o}^R) = (-6,\ 0)\ , \tag{4.19}$$

that is, the RGE running of the $SU(2)_L$ and $SU(2)_R$ gauge couplings are identical.

The existence of the symmetry breaking O' brane allows the localized kinetic terms for the unbroken gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, which spoil the $SU(4)_W$ unification. The most general form of the gauge kinetic term is given by

$$S = \int d^4x dy \left(\frac{1}{4g_5^2}F_{MN}F^{MN} + \delta(y)\frac{1}{4\bar{g}^2}F_{\mu\nu}F^{\mu\nu} + \delta(y-\frac{\pi R}{2})\sum_{i=0}^{2}\frac{1}{4\tilde{g}_i^2}F_{\mu\nu}F^{\mu\nu}\right) \tag{4.20}$$

After integrating out the higher modes, the gauge couplings of the zero modes are

$$\frac{1}{g_i^2} = \frac{\pi R}{2g_5^2} + \frac{1}{\bar{g}^2} + \frac{1}{\tilde{g}_i^2}\ , \tag{4.21}$$

where $g_0 = \sqrt{2}g_{B-L}$, while g_1, g_2 correspond to g_{2L}, g_{2R} coupling respectively. The term $\bar{g}^2$ is irrelevant because it preserves $SU(4)_W$ unification and will not affect the tree-level weak mixing angle. We can assume that the bulk and brane kinetic terms have comparable strength [16] at a cut-off scale Λ (higher than or equal to the unification scale M_U). Since g_5^2 has mass dimension, we can estimate its strength to be $g_5^2\Lambda$ at the cut-off scale, which implies $g_5^2\Lambda \sim \tilde{g_i}^2$. We can see that at tree level the $SU(4)_W$ violating term is suppressed by M_C/Λ and hence the effects can be neglected if $M_C \ll \Lambda$. Besides, it is natural to set such localized gauge kinetic terms to zero at tree level in a fundamental theory. Then for a weakly coupled theory, such localized kinetic terms can only arise at loop level and thus highly suppressed.

The one-loop corrections to the weak mixing angle come from the $SU(4)_W$ violating effects but not from the $SU(4)_W$ conserving effects. For the energy scale in the range $2NM_C < E < (2N+1)M_C$ with $N \gg 1$, we can estimate the RGE running by summing over the contribution of the KK modes. Using Stirling's approximation

$$\ln(N!) \simeq N\ln N - N + \frac{1}{2}\ln(2\pi N)\ , \tag{4.22}$$

and

$$\ln\left[1 \times 3 \times ... \times (2N-1)\right] = \ln\left[(2N)!\right] - N \ln 2 - \ln(N!) \simeq \left(N + \frac{1}{2}\right) \ln 2 + N \ln N - N \ , \tag{4.23}$$

we can write

$$\alpha_i^{-1}(E) \simeq \alpha_i^{-1}(M_R) + \frac{b_i}{2\pi} \ln\left(\frac{M_R}{E}\right) - \frac{1}{4\pi}(b_{i,o} + b_{i,e})\left[\frac{E}{M_C} - \ln 2\right] + \frac{b_{i,e}}{4\pi} \ln\left(\frac{\pi E}{2M_C}\right) \tag{4.24}$$

Thus, after the KK modes contributions are included, the RGE running of the gauge couplings are proportional to $N = E/(2M_C)$, which is a power law running (this agrees with the results of [24]). The relative running of $SU(2)_L$ (identical to $SU(2)_R$) and $U(1)_{B-L}$ is not affected by the $SU(4)_W$ conserving power-law running, instead this running is logarithmic due to $SU(4)_W$ violating effects. In OGSB cases, it is general to have

$$b_{B-L,e} + b_{B-L,o} = b_{L,e} + b_{L,o} = b_{R,e} + b_{R,o} \ , \tag{4.25}$$

which also holds in our case. Due to the universally occurring $b_o + b_e$ term, we can replace b_e with $-b_o$ in the relative running between the gauge couplings. The running of the minimal left-right gauge couplings is given by

$$\frac{1}{g_i^2}(M_R) \simeq \frac{1}{g_*^2}(M_U) + \frac{a}{16\pi^2}\left[\left(\frac{M_U}{M_C}\right) - \ln 2\right] + \frac{\tilde{b}_i}{8\pi^2} \ln\frac{M_U}{M'_C} + \frac{\tilde{c}_i}{8\pi^2} \ln\frac{M'_C}{M_R} \tag{4.26}$$

where $M'_C = 2M_C/\pi$, the coefficient a which is universal and $\tilde{b}_i$ are given in our case by

$$a = b_{i,o} + b_{i,e} \ , \qquad \tilde{b}_i = b_i - \frac{1}{2} b_{i,e} \ , \qquad \tilde{c}_i = b_i \ . \tag{4.27}$$

Then the Weinberg angle for non-SUSY cases is

$$\sin^2\theta_W(M_Z) = \frac{1}{4} - \alpha_{em}(M_Z)\left[\frac{\tilde{b}_1 - \tilde{b}_2}{4\pi} \ln\frac{M_U}{M'_C} + \frac{\tilde{c}_1 - \tilde{c}_2}{4\pi} \ln\frac{M'_C}{M_R} + \frac{d_1 - 3d_2}{8\pi} \ln\frac{M_R}{M_Z}\right] \tag{4.28}$$

where (d_1, d_2) are the one-loop beta functions for $U(1)_Y, SU(2)_L$ in the energy range between M_R and M_Z. The Weinberg angle for SUSY cases is given by

$$\sin^2\theta_W(M_Z) = \frac{1}{4} - \alpha_{em}(M_Z) \left[\frac{\tilde{b}_1 - \tilde{b}_2}{4\pi} \ln\frac{M_U}{M'_C} + \frac{\tilde{c}_1 - \tilde{c}_2}{4\pi} \ln\frac{M'_C}{M_R} + \frac{d_1 - 3d_2}{8\pi} \ln\frac{M_R}{M_S} + \frac{e_1 - 3e_2}{8\pi} \ln\frac{M_S}{M_Z}\right] \ , \tag{4.29}$$

where (d_1, d_2) and (e_1, e_2) denote the one-loop beta functions for $U(1)_Y$ and $SU(2)_L$ for $M_R > E > M_S$ and $M_S > E > M_Z$, respectively.

From the above formulas, we can estimate the unification scale once the compactification scale M'_C is specified. In fact, we can obtain the unification scale more precisely by taking into account each KK-mode contribution step by step. In SUSY left-right unification

cases, we can see from the beta functions that $\tilde{b}_i = 2$ is universal for $SU(2)_L, SU(2)_R$ and $U(1)_{B-L}$. It means that there is no relative running between the three gauge couplings. However, Eq. (4.24) is not valid if the unification scale $M_U \sim NM_C$ satisfies $N \sim \mathcal{O}(1)$ with which the summation approximation Eq. (4.22) is not valid. Thus, we anticipate the unification occurs at the order of the compactification scale if we require that the gauge coupling at unification scale be not strong coupled (weakly coupled unification). We can also identify the unification scale M_U as the cut off scale Λ if the gauge coupling would be strongly coupled at the unification scale. We know that M_S is fixed to be within several hundreds GeV in order to give an explanation of the gauge hierarchy problem by supersymmetry (We will not discuss the non-interesting case of high scale supersymmetry with $M_S > M_R$ here). The detailed numerical calculations show that there is a fairly large parameter space for the values of M_R and M_C in which the weakly coupled unification is possible. We find that the compactification scale M_C is required to be larger than 150 TeV in order to get successful weakly coupled gauge coupling unification. While the larger the M_C, the lower the possible value of M_R that is allowed by the weakly coupled gauge coupling unification. For example, the parameter M_R is required to be larger than 70 TeV with $M_C = 150$ TeV. While if $M_C = 200$ TeV, the allowed M_R can be as low as 40 TeV. Fixing the left-right scale, which is identified as the $SU(2)_R$ gauge boson masses, to $M_R = 100$ TeV, the sfermion mass $M_S = 600$ GeV, and the compactification scale $M_C = 200$ TeV, we obtain

$$\alpha_{B-L}^{-1}(M_R) = 28.810705 \ , \qquad \alpha_L^{-1}(M_R) = \alpha_R^{-1}(M_R) = 28.742994 \ , \tag{4.30}$$

and a weakly coupled unification scale

$$M_U = 323.5 \text{ TeV}. \tag{4.31}$$

Because the compactification scale is relatively high (higher than 150 TeV), the low-energy effective theory is the supersymmetric left-right model.

If the compactification scale M_C is lower than 150 TeV, strongly coupled unification can occur. For example, if we chose TeV-scale extra-dimension with $M_C = 5.0$ TeV while $M_R = 2.0$ TeV, the strongly coupled unification scale (identify as the cut off scale Λ) is $M_U \sim 30 M_C \sim 150$ TeV. We can see that $M_C \sim 0.01\Lambda$ so that the uncertainties from brane kinetic terms are very small.

In non-SUSY cases, the low-energy left-right model contains one bi-doublet, one $SU(2)_L$ triplet and one $SU(2)_R$ triplet. The bulk Higgses contain two $\overline{\mathbf{10}}$ dimensional representations with parity assignments $\eta = 1$ and $\eta' = \pm 1$ [3]. We obtain the following beta functions for the gauge couplings of $SU(2)_L$ and the normalized $U(1)_{B-L}$:

$$(b_1, b_2, b_3) = (\ 7, -3, -7) \qquad \text{for} \ \ M_Z < E < M_R \ , \tag{4.32}$$

$$(b_1, b_2^L, b_2^R, b_3) = \left(\frac{7}{3}, -\frac{7}{3}, -\frac{7}{3}, -7\right) \ \text{for} \ \ M_R < E < M_C \ . \tag{4.33}$$

[3] As noted previously, we cannot introduce a **6** representation Higgs only, because the low energy mass spectrum is not acceptable.

The beta functions of the KK modes are

$$(b_{B-L,e}, b_{B-L,o}) = (1, -13) \ , \tag{4.34}$$

$$(b^L_{2,e}, b^L_{2,o}) = (-6, -6) \ , \tag{4.35}$$

$$(b^R_{2,e}, b^R_{2,o}) = (-6, -6) \ , \tag{4.36}$$

$$(b^R_{3,e}, b^R_{3,o}) = (-21/2, 0) \ . \tag{4.37}$$

The beta functions of $SU(2)_R$ are the same as those of $SU(2)_L$ due to the left-right symmetry.

In the non-SUSY case, the power law running with negative beta functions drive the gauge couplings asymptotically free. We assume here the unification scale M_U is less than the cut off scale Λ. In this case, there are still some allowed parameter space for the values of M_C and M_R which admit gauge coupling unification. In fact, M_C is allowed to be as low as 3.0 TeV with $M_R = 2.2$ TeV (although it is not natural in non-SUSY case to get such low M_R). However, the numerical calculations indicate that the successful gauge coupling unification requires the compactification scale M_C to be lower than 8.0 TeV. For M_C higher than 8.0 TeV, the $SU(2)_L$ and $U(1)_{B-L}$ gauge couplings tend to be zero asymptotically without intersection. Choosing $M_R = 3.0$ TeV and $M_C = 5.0$ TeV, we obtain

$$\alpha_{B-L}(M_R) = 31.6011 \ , \qquad \alpha_L(M_R) = \alpha_R(M_R) = 31.2399 \ , \tag{4.38}$$

and a unification scale much lower than previously

$$M_U = 5.2473 \text{ TeV} \ . \tag{4.39}$$

In this scenario the relatively low left-right and compactification scales allow for a unification scale of several TeV. Although low M_R scenario needs fine-tunning, it is however possible. Such low-energy unification may have numerous interesting phenomenological consequences.

The generic phenomenology of our model is similar to that of any other theories with an extra dimension and thus is not discussed here. But our model has some additional phenomenological features. The scenario predicts the existence of doubly charged gauge bosons at several TeVs which may be within the reach of the LHC. These heavy gauge bosons have gauge couplings to leptons while have no couplings to quarks. Since the $(+,-)$ modes vanish on the O' brane, they can only have derivative couplings to quarks. But two quark interactions with A^X_μ are forbidden because of non-matching quantum numbers. From the mode expansion of the gauge couplings to leptons, which is similar to that of the Yukawa couplings, we can see that the doubly charged heavy gauge boson A^{--} can couple to two charged leptons. It can decay into electron pairs or a pair of $SU(2)_L$ and $SU(2)_R$ charged gauge bosons W_1^- and W_2^-. The coupling of the first KK excitations of the real scalar A^a_5 with the leptons can also give couplings of the charge-two real scalar to charged lepton pairs. We know that ϕ_3 is non-vanishing on the O' brane because it has parity $(-,+)$ under projection. Similar to heavy gauge boson cases, its couplings to two quarks are forbidden because of its non-matching quantum numbers. Our model also have $SU(2)_L$ singlets charged scalars with $B-L = \pm 2$. Such scalars can decay into lepton pairs like $\nu_e \mu$.

5. $SU(4)_W$ Breaking to $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$

As we demonstrated it is advantageous to break the $SU(4)_W$ to the minimal left-right model via orbifolding, and the corresponding OGSB chain for $SU(4)_W$ can be fairly rich. In this section we show that we can break $SU(4)_W$ to $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ which also leads to interesting phenomenology.

In this case, our starting point is again the five dimensional $N = 1$ supersymmetric $SU(3)_C \times SU(4)_W$ gauge symmetry. First, we consider the parity assignments in term of the fundamental representation of $SU(3)_C \times SU(4)_W$:

$$\begin{aligned} P &= \mathrm{diag}(+1, +1, +1) \otimes \mathrm{diag}(+1, +1, +1, -1) \ , \\ P' &= \mathrm{diag}(+1, +1, +1) \otimes \mathrm{diag}(+1, +1, -1, -1) \ . \end{aligned} \tag{5.1}$$

Boundary conditions break $N = 2$ supersymmetry to $N = 1$ in four dimensions. The $SU(3)_C \times SU(3)_W \times U(1)_1$ gauge symmetry is preserved at the O brane while it is broken to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at the O' brane [4]. The zero modes preserve the $SU(3)_C \times SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ gauge symmetry, which can be seen through the form of corresponding generators in $SU(4)_W$.

We introduce two $N = 2$ Higgs hypermultiplets in $SU(3)_C \times SU(4)_W$ symmetric representations in the bulk. These contain the $N = 1$ chiral supermultiplets $\Phi^1(1, 10)$ and $\Phi^2(1, \overline{10})$ as well as their conjugate chiral fields. The parity assignments for the Higgs sector read

$$\eta_{\Phi^i} = 1 \ , \qquad \eta'_{\Phi^i} = -1 \ . \tag{5.2}$$

This leads to the following parity assignments for the Higgs hypermultiplets

$$\begin{aligned} \Phi^i &= \begin{pmatrix} (+,-) & (+,-) & (+,+) & (-,+) \\ (+,-) & (+,-) & (+,+) & (-,+) \\ (+,+) & (+,+) & (+,-) & (-,-) \\ (-,+) & (-,+) & (-,-) & (+,-) \end{pmatrix} \ , \\ (\Phi^i)^c &= \begin{pmatrix} (-,+) & (-,+) & (-,-) & (+,-) \\ (-,+) & (-,+) & (-,-) & (+,-) \\ (-,-) & (-,-) & (-,+) & (+,+) \\ (+,-) & (+,-) & (+,+) & (-,+) \end{pmatrix} \ . \end{aligned} \tag{5.3}$$

The $SU(2)_L$ doublets H_u and H_d arise from the bulk zero modes of Φ^i, and two $SU(2)_L$ singlets T_1 and T_2 from that of $(\Phi^i)^c$.

Fermions can be located at the fix points O or O'. Since at O the gauge symmetry is $SU(3)_C \times SU(3)_W \times U(1)_1$, if we place all the matter on the O brane, we have to introduce mirror fermions for quarks similarly to the 3-3-1 model. Thus, the most economical way is

[4] In fact, various combinations of $U(1)$ Abelian groups may remain, since the inner automorphism OGSB will not reduce the rank of the groups while Z_n orbifolding [11]. For example, the $U(1)_1$ and the diagonal components T^8 of $SU(3)_W$ can be recombined into $U(1)_{B-L}$ and $U(1)_{3R}$.

to locate all matter at the O' brane (although it is also possible to put leptons on the O brane while quarks are on O' brane). Since at O' only the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry is preserved, we can start with a left-right gauge invariant Lagrangian and then integrate out the heavy modes to get the $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ Lagrangian in four dimension.

The matter content at the O' brane can be that of the minimal left-right model:

$$(\mathbf{3},\mathbf{2},\mathbf{1}) : Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} , \qquad (\bar{\mathbf{3}},\mathbf{1},\bar{\mathbf{2}}) : Q^c_L = \begin{pmatrix} u^c_L \\ d^c_L \end{pmatrix} , \tag{5.4}$$

$$(\mathbf{1},\mathbf{2},\mathbf{1}) : L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} , \qquad (\mathbf{1},\mathbf{1},\bar{\mathbf{2}}) : L^c_L = \begin{pmatrix} \nu^c_L \\ e^c_L \end{pmatrix} . \tag{5.5}$$

In terms of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ representations the Higgses can be written as:

$$\Phi^1(\mathbf{10}) = \begin{pmatrix} A_{2\times 2} & \phi_{2\times 2} \\ \phi_{2\times 2} & B_{2\times 2} \end{pmatrix} , \qquad \Phi^2(\bar{\mathbf{10}}) = \begin{pmatrix} A'_{2\times 2} & \phi'_{2\times 2} \\ \phi'_{2\times 2} & B'_{2\times 2} \end{pmatrix} . \tag{5.6}$$

$$(\Phi^1)^c(\overline{\mathbf{10}}) = \begin{pmatrix} A_1 & S_1 & D_{11} & D_{12} \\ S_1 & A_2 & D_{21} & D_{22} \\ D_{11} & D_{21} & A_3 & T_1 \\ D_{12} & D_{22} & T_1 & A_4 \end{pmatrix}, \quad (\Phi^2)^c(\mathbf{10}) = \begin{pmatrix} B_1 & S_2 & E_{11} & E_{12} \\ S_2 & B_2 & E_{21} & E_{22} \\ E_{11} & E_{21} & B_3 & T_2 \\ E_{12} & E_{22} & T_2 & B_4 \end{pmatrix}. \tag{5.7}$$

The parity of the brane fields are determined by the requirement that all the gauge invariant operators on the O' brane must transform covariantly under P parity which is the consequence of the identification of the $y = \pi R/2$ and $y = -\pi R/2$ branes. From the kinetic terms and parity assignments follows the parity of the matter content on the O' brane:

$$\begin{aligned} &P: \; Q_L = \pm(+,+) , && P: \; L_L = \pm(+,+) , \\ &P: \; Q^c_L = \pm(+,-) , && P: \; L^c_L = \pm(+,-) . \end{aligned} \tag{5.8}$$

From the parity properties of gauge invariant operators on the O' brane (which we do not list here) the Yukawa couplings of the bulk Higgses to the brane fermions can be obtained

$$\begin{aligned} \mathcal{L}_5 = &\int d^2\theta \frac{\sqrt{\pi R}}{2} \\ &\times \Big\{ \; \Big[\delta(y - \frac{\pi R}{2}) \pm \delta(y + \frac{\pi R}{2})\Big] \sum_{ij} \Big(Y^1_{ij} \epsilon^{ab} (Q_L)^i_a \, (Q^c_L)^j_c \, (\phi)^c_b + Y^2_{ij} \epsilon^{bc} (Q_L)^i_a (Q^c_L)^j_c (\phi')^a_b \Big) \\ &+ \frac{1}{2} \Big[\delta(y - \frac{\pi R}{2}) \pm \delta(y + \frac{\pi R}{2})\Big] \sum_{ij} \big(Y^3_{ij} \epsilon^{ab} (L_L)^i_a (L^c_L)^j_c (\phi)^c_b + Y^4_{ij} \epsilon^{bc} (L_L)^i_a (L^c_L)^j_c (\phi')^a_b \big) \\ &+ \frac{1}{2} \Big[\delta(y - \frac{\pi R}{2}) + \delta(y + \frac{\pi R}{2})\Big] \sum_{ij} \Big(Y^5_{ij} \epsilon^{ab} (L_L)^i_a (L_L)^j_b T_1 + Y^6_{ij} \epsilon^{ab} (L^c_L)^i_a (L^c_L)^j_b T_2 \Big) \; \Big\} \end{aligned} \tag{5.9}$$

Here the $\pm$ signs correspond to the relative parity assignment (identical or inverse) in front of Q_L and Q^c_L (L_L and L^c_L), respectively. After expanding ϕ and ϕ^c in their KK modes,

we can see that amongst the zero modes only two $SU(2)_L$ doublets remain which are identified as H_u and H_d of the supersymmetric standard model.The electric charged field $T_1(T_2)$ can couple to two leptons as $\nu_L e_L(\nu_R e_R)$ etc. The $U(1)_{B-L}$ number for quarks and leptons can be determined by anomaly cancellation requirements for $[SU(2)_L]^2 U(1)_{B-L}$, $[SU(2)_R]^2 U(1)_{B-L}$, $[U(1)_{B-L}]^3$, as well as $[\text{Gravity}]^2 U(1)_{B-L}$, and $[SU(3)_C]^2 U(1)_{B-L}$, etc. The normalization of the Higgs sector can be determined by the requirement that the Yukawa couplings should be invariant under $U(1)_{B-L}$. The charge quantization conditions, in terms of the $SU(4)_W$ fundamental representation, are

$$Q_1 + Q_3 = Q_2 + Q_3 = 0, \qquad Q_3 + Q_4 = 2b \ , \tag{5.10}$$

where b is the $U(1)_{B-L}$ number for leptons. The fields T_1 and T_2 are necessary to determine the $U(1)_{B-L}$ quantization conditions because they give the second equation in the previous formula. From the first equation and traceless condition follows that the $U(1)_{B-L}$ generator is proportional to the $SU(4)_W$ generator

$$T_{B-L} \ = \ \text{diag}(-a, -a, \ a, \ a) \ . \tag{5.11}$$

From the second equation we obtain that $a = b = 1$. (Here we rely on the phenomenological requirement that the relative normalization of the $U(1)_{B-L}$ charge between the Higgs and lepton sectors was chosen to be $b = 1$). From these quantization conditions, we obtain $2g^2_{B-L} = g^2_4$. Since the $U(1)_{3R}$ gauge group can be realized as the diagonal subgroup of $SU(2)_R$, its normalization condition is set by $SU(2)_R$, which leads to the relation $g^2_{3R} = g^2_4$. From the charge assignments we obtain

$$Q = T_{3L} + \frac{Y_{3R}}{2} + \frac{Y_{B-L}}{2} \ . \tag{5.12}$$

The tree level weak mixing angle is again $\sin^2\theta_W = 0.25$.

The quantization conditions imply the parity and quantum numbers for all the bulk fields

$$\begin{aligned}
V(\mathbf{15}) &= \mathbf{3}^{(+,+)}_{\mathbf{(0,0)}} \oplus \mathbf{1}^{(+,+)}_{\mathbf{(0,0)}} \oplus \mathbf{1}^{(+,+)}_{\mathbf{(0,0)}} \oplus \mathbf{2}^{(+,-)}_{\mathbf{(-1,-2)}} \oplus \bar{\mathbf{2}}^{(+,-)}_{\mathbf{(1,2)}} \oplus \mathbf{2}^{(-,-)}_{\mathbf{(1,-2)}} \oplus \bar{\mathbf{2}}^{(-,-)}_{\mathbf{(-1,2)}} \oplus \mathbf{1}^{(-,+)}_{\mathbf{(-2,0)}} \oplus \mathbf{1}^{(-,+)}_{\mathbf{(2,0)}} \\
\Sigma(\mathbf{15}) &= \mathbf{3}^{(-,-)}_{\mathbf{(0,0)}} \oplus \mathbf{1}^{(-,-)}_{\mathbf{(0,0)}} \oplus \mathbf{1}^{(-,-)}_{\mathbf{(0,0)}} \oplus \mathbf{2}^{(-,+)}_{\mathbf{(-1,-2)}} \oplus \bar{\mathbf{2}}^{(-,+)}_{\mathbf{(1,2)}} \oplus \mathbf{2}^{(+,+)}_{\mathbf{(1,-2)}} \oplus \bar{\mathbf{2}}^{(+,+)}_{\mathbf{(-1,2)}} \oplus \mathbf{1}^{(+,-)}_{\mathbf{(-2,0)}} \oplus \mathbf{1}^{(+,-)}_{\mathbf{(2,0)}} \\
\Phi(\mathbf{10}) &= \mathbf{3}^{(+,-)}_{\mathbf{(0,-2)}} \oplus \mathbf{2}^{(+,+)}_{\mathbf{(1,0)}} \oplus \mathbf{2}^{(-,+)}_{\mathbf{(-1,0)}} \oplus \mathbf{1}^{(+,-)}_{\mathbf{(2,2)}} \oplus \mathbf{1}^{(+,-)}_{\mathbf{(-2,2)}} \oplus \mathbf{1}^{(-,-)}_{\mathbf{(0,2)}}, \\
\Phi^c(\overline{\mathbf{10}}) &= \bar{\mathbf{3}}^{(-,+)}_{\mathbf{(0,2)}} \oplus \bar{\mathbf{2}}^{(-,-)}_{\mathbf{(-1,0)}} \oplus \bar{\mathbf{2}}^{(+,-)}_{\mathbf{(1,0)}} \oplus \mathbf{1}^{(-,+)}_{\mathbf{(-2,-2)}} \oplus \mathbf{1}^{(-,+)}_{\mathbf{(2,-2)}} \oplus \mathbf{1}^{(+,+)}_{\mathbf{(0,-2)}}, \\
\Phi(\overline{\mathbf{10}}) &= \bar{\mathbf{3}}^{(+,-)}_{\mathbf{(0,2)}} \oplus \bar{\mathbf{2}}^{(+,+)}_{\mathbf{(-1,0)}} \oplus \bar{\mathbf{2}}^{(-,+)}_{\mathbf{(1,0)}} \oplus \mathbf{1}^{(+,-)}_{\mathbf{(-2,-2)}} \oplus \mathbf{1}^{(+,-)}_{\mathbf{(2,-2)}} \oplus \mathbf{1}^{(-,-)}_{\mathbf{(0,-2)}}, \\
\Phi^c(\mathbf{10}) &= \mathbf{3}^{(-,+)}_{\mathbf{(0,-2)}} \oplus \mathbf{2}^{(-,-)}_{\mathbf{(1,0)}} \oplus \mathbf{2}^{(+,-)}_{\mathbf{(-1,0)}} \oplus \mathbf{1}^{(-,+)}_{\mathbf{(2,2)}} \oplus \mathbf{1}^{(-,+)}_{\mathbf{(-2,2)}} \oplus \mathbf{1}^{(+,+)}_{\mathbf{(0,2)}}.
\end{aligned} \tag{5.13}$$

Subscripts denote $U(1)_{3R}$ and $U(1)_{B-L}$ quantum numbers, respectively. We can see that there are zero mode components in $\Sigma(\mathbf{15})$ decompositions. Such zero modes can act as Higgs doublets in the MSSM, if we adopt the gauge-Higgs unification scheme. However such Higgs fields cannot couple to matter fields because of un-matching quantum numbers.

The $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ gauge symmetry can be broken to the SM one (in SUSY cases) via the bulk Higgs fields $H^1(\mathbf{1}, \mathbf{4})$ and $H^2(\mathbf{1}, \bar{\mathbf{4}})$ (here $SU(3)_C \times SU(4)_W$

representations are shown). Parity can be assigned to these Higgses as

$$\eta_{H^i} = -1 \ , \qquad \eta'_{H^i} = -1 \qquad (i = 1, 2) \ . \tag{5.14}$$

From the decomposition of $SU(4)_W$ in terms of $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$

$$\begin{aligned} (H^1)(\mathbf{4}) &= \mathbf{2}^{(-,-)}_{(0,-1)} \oplus \mathbf{1}^{(-,+)}_{(1,1)} \oplus \mathbf{1}^{(+,+)}_{(-1,1)} \ , \quad (H^1)^c(\bar{\mathbf{4}}) = \bar{\mathbf{2}}^{(+,+)}_{(0,1)} \oplus \mathbf{1}^{(+,-)}_{(-1,-1)} \oplus \mathbf{1}^{(-,-)}_{(1,-1)} \ , \\ (H^2)(\bar{\mathbf{4}}) &= \bar{\mathbf{2}}^{(-,-)}_{(0,1)} \oplus \mathbf{1}^{(-,+)}_{(-1,-1)} \oplus \mathbf{1}^{(+,+)}_{(1,-1)} \ , \quad (H^2)^c(\mathbf{4}) = \mathbf{2}^{(+,+)}_{(0,-1)} \oplus \mathbf{1}^{(+,-)}_{(1,1)} \oplus \mathbf{1}^{(-,-)}_{(-1,1)} \ , \end{aligned} \tag{5.15}$$

follows that the zero modes of H^i $(i = 1, 2)$ contain two $SU(2)_L$ singlets $U^1_{(\mathbf{-1},\mathbf{1})}$ and $U^2_{(\mathbf{1},\mathbf{-1})}$ (subscripts denote $U(1)_{3R} \times U(1)_{B-L}$ quantum numbers) which are electrically neutral and cannot couple to matter directly. The zero modes for $(H^i)^c$ contain two Higgs doublets $\bar{\mathbf{2}}^{(+,+)}_{(0,1)}$ and $\mathbf{2}^{(+,+)}_{(0,-1)}$ which can not couple to matter either because of non-matching quantum numbers. After U^1 and U^2 acquire VEVs, the remaining gauge symmetry is broken to the SM gauge group [5]. Note that T_1 and T_2 cannot be used to break this gauge symmetry because they have electric charges.

The beta functions of the gauge couplings $U(1)_Y,\ SU(2)_L,\ SU(3)_C$ read

$$(b_1, b_2, b_3) = \left(\frac{25}{3}, -\frac{7}{3}, -7\right) \qquad \text{for} \quad M_Z < E < M_S \ , \tag{5.16}$$

$$(b_1, b_2, b_3) = (\ 15,\ \ 3, -3) \qquad \text{for} \quad M_S < E < M'_Z \ . \tag{5.17}$$

In the SUSY and SUSY decoupling limits, there are six Higgs doublets. For the $\sqrt{2}U(1)_{B-L}$, $U(1)_{3R}/2$, $SU(2)_L$, and $SU(3)_C$ gauge couplings the beta functions are

$$(b_1^{B-L}, b_1^{3R}, b_2^L, b_3) = \left(\frac{23}{4}, \frac{17}{2},\ 3, -3\right) \quad \text{for} \quad M'_Z < E < M_C \ . \tag{5.18}$$

The beta functions corresponding to the even and odd KK modes at one loop are

$$(b_{B-L,e}, b_{B-L,o}) = (-\frac{1}{2}, \frac{13}{2}) \ , \tag{5.19}$$

$$(b_{3R,e}, b^L_{3R,o}) = (\ 1,\ 5) \ , \tag{5.20}$$

$$(b^R_{2,e}, b^R_{2,o}) = (-2,\ 8) \ , \tag{5.21}$$

$$(b^R_{3,e}, b^R_{3,o}) = (-6,\ 0) \ . \tag{5.22}$$

Just as in the previous case, the $SU(4)_W$ preserved $b_o + b_e$ is constant for the three gauge couplings. Thus, the relative RGE running between the three gauge couplings are logarithmic. As we do not know the g_{B-L} or g_{3R} gauge couplings at the M'_Z scale (or the relations between the two gauge couplings), we must invoke further assumptions related to

[5] It is also possible to break the remaining gauge group to the SM via localized Higgs fields $A(\mathbf{1}, \mathbf{2}, \mathbf{-1})$ (in terms of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum number) on the O' brane. Such localized brane Higgs fields can break the gauge group $SU(2)_R \times U(1)_{B-L}$ on the O' brane to $U(1)_Y$, which corresponds to breaking the bulk $U(1)_{B-L} \times U(1)_{3R}$ to $U(1)_Y$. The other possibility is to introduce two $\Delta(\overline{\mathbf{10}})$ representations for $SU(4)_W$ with parity assignment $\eta_{\Delta^i} = \eta'_{\Delta^i} = 1$. The VeV of the neutral component $\mathbf{1}^{(+,+)}_{(\mathbf{2},\mathbf{-2})}$ will break $U(1)_{3R}$ as well as give Majorana neutrino masses for right handed neutrino.

them to predict the unification scales. SUSY breaking can again be achieved by the Scherk-Schwarz mechanism through boundary conditions. The tree-level gaugino and Higgsino masses acquired this way will induce loop-level squark and slepton masses.

The phenomenology of this symmetry breaking chain shares many common features with that of the previous cases. For example, there are charge two heavy gauge bosons and two $SU(2)_L$ charged gauge singlets scalars which can only derivatively couple to charged lepton pairs. At energies well below the M_C scale, the low energy effective theory reduces to supersymmetric $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$. This $U(1)$ extension of the SM has been widely studied. The special feature of this scenario is the existence of Higgs doublets which have no tree level couplings to SM fermions even when the low energy $SU(2)_L \times U(1)_Y$ quantum number allow such couplings. The electrically neutral $SU(2)_L$ singlet Higgses, U^1 and U^2, which break the remaining group to the SM, can be viable dark matter candidates.

Alternative Models: We can locate the SM quarks and right-handed charged leptons on the O' brane while placing the SM lepton doublets and right-handed neutrinos in the bulk. We can introduce mirror leptons X_L, X_L^c, $\bar{X}$, $\bar{X}^c$, Y_L, and Y_L^c to fill the bulk hypermultiplets F_i $(i = 1, 2)$ in the $(\mathbf{1}, \mathbf{4})$ representation under $SU(3)_C \times SU(4)_W$:

$$\begin{aligned} F_1 &= (L_L \ X_L) \ , \qquad F_2 = (\bar{X}, (L_L^c)') \ , \\ F_1^c &= (Y_L \ X_L^c) \ , \qquad F_2^c = (\bar{X}^c, Y_L^c) \ . \end{aligned} \tag{5.23}$$

Here $(L_L^c)'$ denotes $(E_L^c \ -\nu_L^c)$, with E_L^c being a charged mirror lepton. Then, we can assign parities as

$$\eta_{F_1} = 1 \ , \quad \eta'_{F_1} = 1 \ , \quad \eta_{F_2} = -1 \ , \quad \eta'_{F_2} = -1 \ . \tag{5.24}$$

The left-handed leptons and neutrinos L_L arise from F_1, and the right-handed neutrinos from F_2. Note that we cannot fit right-handed leptons in F_1^c because that does not yield the correct quantum numbers. Mirror fermions associated with each SM leptons, except with the right-handed charged leptons, will survive the projection.

As previously, we can locate the SM quarks and right-handed neutrinos on the O' brane while having the SM lepton doublets and right-handed charged leptons in the bulk. The parity assignments read

$$\begin{aligned} P &= \mathrm{diag}(+1, +1, +1) \otimes \mathrm{diag}(+1, +1, -1, +1) \ , \\ P' &= \mathrm{diag}(+1, +1, +1) \otimes \mathrm{diag}(+1, +1, -1, -1) \ . \end{aligned} \tag{5.25}$$

Similarly to our previous case, we obtain mirror fermions associated with each SM leptons from zero modes, except for right-handed neutrinos.

6. Conclusions

In this paper, we propose a low scale $SU(4)_W$ unification model which has two symmetry breaking chains. In the first chain $SU(4)_W$ is broken into the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

minimal left-right model through $S^1/(Z_2 \times Z_2)$ orbifolding. Leptons are fitted into $SU(4)_W$ multiplets and located on a symmetry preserving O brane, while quarks are placed on O' brane where the symmetry is broken. This approach predicts $\sin^2\theta_W = 0.25$ for the weak mixing angle at tree level and leads to a rather low weakly coupled unification scale of order 10^2 TeV with supersymmetry, or as low as several TeV in the non-supersymmetric case. If we introduce mirror fermions and put quarks in the bulk, the model gives a large weak mixing angle $\sin^2\theta_W = 0.45$ which will lead to high-energy unification. The other symmetry breaking chain with the low-energy gauge group $SU(2)_L \times U(1)_{3R} \times U(1)_{B-L}$ after OGSB can also give rise to a weak mixing angle $\sin^2\theta_W = 0.25$ at tree level. In this scenario, leptons and quarks are placed on the O' brane (with broken symmetry) and the quantization conditions are determined by anomaly cancelation requirements. These low-scale unification theories have interesting phenomenological consequences.

One may worry if there are cosmological difficulties associated with this scenario such as the monopole problems etc. In fact there are no monopole problems in our scenario because we break the gauge symmetry via orbifolding. In general there are monopole problems if a gauge symmetry is broken to a subgroup containing $U(1)$ via Higgs mechanism with the unification scale lower than the inflation scale and at the same time higher than TeV scale [29]. It is not a problem in OGSB scenario because the gauge symmetry is broken via boundary conditions with the symmetry broken explicitly in the orbifold fix points. So our scenario is not bothered by the cosmological monopole problems.

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